GoldenGate Journal of Science & Technology 2017, 3: 22 - 26 ©GJST
ISSN 2505-0656 (Print)

Physics

ANALYZING THE COMPONENTS OF LIGHTNING ELECTRIC FIELD

Pitri Bhakta Adhikari
Department of Physics, Tri-Chandra Campus, T.U.

Keywords – *Maxwell's equation; Jefimenko equation; Lightning electric field; Components of lightning electric field*

Abstract

The electric field produced by lightning discharge is the most important aspect of lightning. Analyzing its components helps in a better understanding of the electric field generated during lightning discharge. The practicality of the Jefimenko equations in computations of electric fields, from lightning, was analyzed. The general Jefimenko equations are derived from Maxwell's equation. Different processes for the computations of electric fields due to lightning was compared with the generalized three dimensional Jefimenko equations. The variations in the components electric field with time were analyzed and the individual effects of these components in the total electric field were described.

1. Introduction

A sudden electrostatic discharge that occurs between electrically charged regions between two clouds, cloud and air, and between a cloud and a ground is generally referred as lightning. Bazelyan and Raizer, (2000) described that all lightning discharges can be classified, even without photography, into two groups – inter-cloud discharges and ground strikes. The frequency of the former is two or three times higher than that of the latter. In order for an electric discharge to occur two conditions are of utmost importance, they are a sufficiently high electric potential between two regions of space and a high resistance medium must obstruct the equalization of the opposite charges. As the thundercloud moves over the surface of the earth considered as an infinitely conducting grounded plane, an equal electric charge of opposite polarity is induced on the earth surface. The oppositely charged create an electric field within the air between them. The greater the accumulation of charges, higher will be the electric field. The electric field created due to lightning is a complex process to comprehend. Describing the components of electric field is of utmost importance because of its applications in various fields other than lightning discharges. Thottappillil and Rakov, (2001), have obtained the electric field expression in the form of electro static, induction, and radiation fields. These components are not only important from the lightning discharge point of view but also due individual effects created by them in the surroundings. Nickolaenko and Hayakawa, (1998) described that quasi-static, induction, and radiation field components of severe stroke may trigger modifications in the atmosphere. Thottappillil, (2003) described that static component is dependent on the term containing r^{-3}, the induction component is dependent on the term containing r^{-2}, and the radiation component is dependent on the term containing r^{-1}. Chalmers, (1965) described that the induction component of lightning is a consequence of the retarded potential of varying dipole. Taylor, (1963) found out that the electromagnetic radiation from lightning discharge is in the form of radio energy. So, many researchers have tried to analyze these components of electric field by studying atmosphere and natural disturbances brought by lightning. Thottappillil and Rakov, (2001) has compared different approaches for the computation of lightning electric field. These approaches are traditional dipole Lorentz condition technique and two versions of monopole continuity equation technique. His comparison shows that no matter what the math for electric field, the electric field due to lightning generally consists of three components (ie, electrostatic, induction, and radiation). The use of Jefimenko equations in computation of electric fields due to lightning discharge has gained popularity at recent times. Especially, the three dimensional generalized Jefimenko electric field equation has provided a dynamic way of analyzing the components of electric field. The electric field components of Shao, (2016) generalized three dimensional Jefmenko equation are compared graphically with Thottappillil electric field equation. The interference problem produced by lightning is due to the radiation field of lightning. The radiated electric field from lightning channel contains a vertical and horizontal component of the electric fields that is responsible for producing transients in power lines, telecommunication lines etc. With the help of Jefimenko equations these components of the electric field have been described in this paper.

2. Jefimenko electric field equations and retarded potential

Jefimenko equations, in the field of electromagnetism, describe the behavior of the electric and magnetic fields in terms of the charge and current distribution at retarded times. According to Shao, (2016), Jefimenko equations are pair of equations that relate the electromagnetic fields in an observer's time frame to the charge and current sources in the retarded time frame. Jefimenko equations are the solutions of the Maxwell's equations for given electric charges and currents with the assumptions that only the charges and currents produce electric fields. The electric **(E)** from Jefimenko equations due to the arbitrary charge or current distribution of charge density (ρ) and the current density (**J**) is given as follows.

$$\boldsymbol{E}(\boldsymbol{r},t) = \frac{1}{4\pi\varepsilon_\circ}\int\left[\left(\frac{\rho(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|^3}+\frac{1}{|\mathbf{r}-\mathbf{r}'|^2 c}\frac{\partial\rho(\mathbf{r}',t_r)}{\partial t}\right)(\boldsymbol{r}-\boldsymbol{r}') - \frac{1}{|\mathbf{r}-\mathbf{r}'|c^2}\frac{\partial \boldsymbol{J}(\mathbf{r}',t_r)}{\partial t}\right] d^3\boldsymbol{r}' \quad (1)$$

Where **r'** is a point in the charge distribution, **r** is a point in space, and $t_r = t - \frac{|\boldsymbol{r}-\boldsymbol{r}'|}{c}$, is the retarded time also r $=|\boldsymbol{r}-\boldsymbol{r}'|$.

This equation is the time dependent generalizations of coulombs law to electrodynamics. Heald and Griffiths, (1991), described that while time dependent potentials are simply the retarded forms of static potentials the time dependent fields are more than the retarded forms of coulombs law. Jefimenko equations not only hold for the time varying charges and currents but also when the charges and current distribution are static. Over recent period of time Jefimenko equations has earned a lot of popularity because of its applications other than lightning discharges. Some of these include electromagnetic calculation of nuclear explosions also.

The Jefimenko equation becomes incomplete without the concept of retarded potentials. In electrodynamics, the retarded potentials are those electromagnetic potentials for the electric field generated by time-varying electric current or charge distribution in the past. As the electromagnetic information travels at the speed of light an observer at a distant from the charge distribution gets the "information" of earlier times and earlier position. In non-static case therefore, it's not the status of the source at present that matters, but rather, its position at some earlier time t_r, known as the retarded time, after the "message" left. Since the electromagnetic waves travel at the speed of light, the delay in travelling any distance, $|\boldsymbol{r}-\boldsymbol{r}'|$ is $\frac{|\boldsymbol{r}-\boldsymbol{r}'|}{c}$ hence, $t_r = t - \frac{|\boldsymbol{r}-\boldsymbol{r}'|}{c}$.

For time dependent fields, the scalar potential V(**r**, t) and vector potential **A**(**r,** t) as described by Griffiths, (1999) is as follows,

$$\boldsymbol{V}(\boldsymbol{r},t) = \frac{1}{4\pi\varepsilon_\circ}\int\frac{\rho(\boldsymbol{r}',t_r)}{|\boldsymbol{r}-\boldsymbol{r}'|}d\tau' \quad (2)$$

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_\circ}{4\pi}\int\frac{\boldsymbol{J}(\boldsymbol{r}',t_r)}{|\boldsymbol{r}-\boldsymbol{r}'|}d\tau' \quad (3)$$

Where $\rho(\boldsymbol{r}',t_r)$ is the charge density that prevailed at point **r'** at the retarded time t_r and τ is less than or equals to time t. Griffiths, (1999), described that because the integrals are evaluated at retarded time they are called retarded potentials. The electric field **E**(**r**, t) can be calculated from $V(\boldsymbol{r},t)$ and $\boldsymbol{A}(\boldsymbol{r},t)$ which eventually leads to Jefimenko electric field equations given below in unit **3.**

3. Jefimenko electric field equations from Maxwell's equation

Maxwell's equations are a set of partial differential equations that describe how electric and magnetic fields are generated by charges and currents. The major consequence of these equations is that they visualize how varying electric and magnetic fields propagate at the speed of light. These equations are named after the physicists and mathematician James Clerk Maxwell. He published an early form of these equations and first purposed the electromagnetic phenomenon of light. Maxwell's equations represent one of the most elegant and concise ways to state the fundamentals of electricity and magnetism.

Volume charge density (ρ) and Volume current density (**J**) generate the electric and magnetic field. Let **E**(**r, t**) and **B**(**r**, t) are the electric and magnetic field intensity, $\mu_\circ$ and $\varepsilon_\circ$ are the permeability and permittivity. We know the Maxwell's equations are:

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r},t) = \frac{\rho(\boldsymbol{r}',t_r)}{\varepsilon_\circ} \quad (4)$$

$$\nabla \cdot \boldsymbol{B}(\boldsymbol{r},t) = 0 \quad (5)$$

$$\nabla \times \boldsymbol{E}(\boldsymbol{r},t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r},t)}{\partial t} \quad (6)$$

$$\nabla \times \boldsymbol{B}(\boldsymbol{r},t) = \mu_\circ \boldsymbol{J}(\boldsymbol{r}',t_r) + \frac{1}{c^2}\frac{\partial \boldsymbol{E}(\boldsymbol{r},t)}{\partial t} \quad (7)$$

Where c is the speed of light which relates $c^2 = \frac{1}{\mu_\circ \varepsilon_\circ}$

So the electric and magnetic fields are $\boldsymbol{E}(\boldsymbol{r},t) = -\nabla V(\boldsymbol{r},t)$ *and* $\boldsymbol{B}(\boldsymbol{r},t) = \nabla \times \boldsymbol{A}(\boldsymbol{r},t)$. Hence,

$$\nabla \times \boldsymbol{E}(\boldsymbol{r},t) + \frac{\partial \boldsymbol{A}(\boldsymbol{r},t)}{\partial t} = 0$$

$$\nabla^2 V(\boldsymbol{r},t) + \frac{\partial \nabla \cdot \boldsymbol{A}(\boldsymbol{r},t)}{\partial t} = -\frac{\rho(\boldsymbol{r}',t_r)}{\varepsilon_\circ} \quad (8)$$

Substituting equations of $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{B}(\boldsymbol{r},t)$ into Faraday's law and as the term whose curl gives zero can be written as the gradient of scalar so we obtain equation (8). Where, t_r is retarded time.

Again from the Maxwell's forth relation,

$$\nabla^2 A(r,t) - \mu_\circ\varepsilon_\circ \frac{\partial^2 A(r,t)}{\partial t^2} - \nabla\left(\nabla \cdot A(r,t) + \mu_\circ\varepsilon_\circ \frac{\partial V(r,t)}{\partial t}\right) = -\mu_\circ J(r',t_r) (9)$$

These two equations (8) and (9) contain all the information in Maxwell's equation.

For the non static cases,

$$V(\boldsymbol{r},t) = \frac{1}{4\pi\varepsilon_\circ}\int \frac{\rho(\boldsymbol{r}',t_r)}{r} d\tau \text{ and } \boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_\circ}{4\pi}\int \frac{\boldsymbol{J}(\boldsymbol{r}',t_r)}{r} d\tau$$

Where $\rho(\boldsymbol{r}',t)$ is the charge density that prevailed at point **r'** at the retarded time where

$$t_r = t - \frac{|\boldsymbol{r} - \boldsymbol{r}'|}{c}$$

We know from the Lorentz condition,

$$\nabla \cdot \boldsymbol{A}(\boldsymbol{r},t) + \mu_\circ\varepsilon_\circ \frac{\partial V(\boldsymbol{r},t)}{\partial t} = 0$$

For any scalar function addition or subtraction is possible, due to this, there is no effect on the electric and magnetic field (E and B). Griffiths, (1999), described such changes in scalar potentials are called gauge transformation. Hence the gradient of scalar potential gives,

$$\nabla V(\boldsymbol{r},t) = \frac{1}{4\pi\varepsilon_\circ}\int \left[-\frac{\dot{\rho}(\mathbf{r}',t_r)}{c}\frac{\hat{\boldsymbol{r}}}{r} - \rho(\boldsymbol{r}',t_r)\frac{\hat{\boldsymbol{r}}}{r^2}\right] d\tau$$

By substituting

$$\nabla\rho(\boldsymbol{r}',t_r) = \dot{\rho}(\mathbf{r}',t_r)\ \nabla t_r = -\frac{1}{c}\dot{\rho}(\mathbf{r}',t_r)\nabla r, \nabla r = \hat{\boldsymbol{r}} \text{ and}$$

$$\nabla\frac{1}{r} = -\frac{\hat{\boldsymbol{r}}}{r^2}, \quad \hat{\boldsymbol{r}} = \frac{\mathbf{r}}{r}$$

On taking the divergence of the gradient of scalar potential we know,

$$\nabla^2 V(\boldsymbol{r},t) = \frac{1}{4\pi\varepsilon_\circ}\int \left[\frac{1}{c^2}\frac{\ddot{\rho}(\mathbf{r}',t_r)}{r} - 4\pi\rho(\boldsymbol{r}',t_r)\delta^3(\hat{\boldsymbol{r}})\right] d\tau$$

$$\nabla^2 V(\boldsymbol{r},t) = \frac{1}{c^2}\frac{\partial^2 V(\boldsymbol{r},t)}{\partial t^2} - \frac{1}{\varepsilon_\circ}\rho(\boldsymbol{r}',t_r)$$

By substituting

$$\nabla\dot{\rho}(\mathbf{r}',t_r) = -\frac{1}{c}\ddot{\rho}(\mathbf{r}',t_r)\nabla r, \nabla\cdot\frac{\hat{\boldsymbol{r}}}{r} = \frac{1}{r^2} \quad \nabla\cdot\frac{r}{r^2} = 4\pi\delta^3(\hat{\boldsymbol{r}})$$

Where, $\delta^3(\hat{\boldsymbol{r}})$ is a three dimensional Dirac delta function. The time derivative of scalar potential A is $\frac{\partial A(r,t)}{\partial t} = \frac{\mu_\circ}{4\pi}\int \frac{\dot{j}(r',t_r)}{r} d\tau$

Hence,

$$\boldsymbol{E}(\boldsymbol{r},t) = \frac{1}{4\pi\varepsilon_\circ}\int \left[\frac{\rho(\mathbf{r}',t)}{r^2}\hat{\boldsymbol{r}} + \frac{\dot{\rho}(\mathbf{r}',t)}{cr}\hat{\boldsymbol{r}} - \frac{\dot{\mathbf{j}}(\boldsymbol{r}',t_r)}{rc^2}\right] d\tau \quad (10)$$

This is the time dependent generalization of coulombs law. Hence, equation (10) is known as the Jefimenko electric field equation derived from Maxwell's equation.

4. Electric fields of lightning discharges from Jefimenko's equation

A general method of calculation of the time dependent electromagnetic fields was given by Lorentz, (1867) in which the retarded potentials were first introduced. These are

$$\emptyset(\boldsymbol{x},t) = \int \frac{\rho(\boldsymbol{x}',t_r)}{|\boldsymbol{x} - \boldsymbol{x}'|} d\boldsymbol{x}' \quad (11)$$

$$\boldsymbol{A}(\boldsymbol{x},t) = \frac{\mu_\circ}{4\pi}\int \frac{\boldsymbol{J}(\boldsymbol{x}',t_r)}{|\boldsymbol{x} - \boldsymbol{x}'|} d\boldsymbol{x}' \quad (12)$$

Where $\emptyset$ and **A** are the scalar and vector potentials in Gaussian units, ρ and **J** are the charge and current densities. According to McDonald, (1997), Lorentz didn't explicitly display the electric field **E** and magnetic field **B**, though he noted they could be obtained from equations,

$$\boldsymbol{E} = -\boldsymbol{\nabla}\emptyset - \frac{1}{C}\frac{\partial \boldsymbol{A}}{\partial t}$$

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

He further explained that had Lorentz work been better received by Maxwell, the time dependent electric and magnetic fields have been known a century ago because Maxwell couldn't understand the prospect that time-dependent potentials were useful tools and hence deserved models of their own.

In the computation of electromagnetic field due to lightning discharges Uman et al., (1975) derived a pair of time dependent electro-magnetic field equation for a one dimensional current element given by

$$d\boldsymbol{E}(\boldsymbol{r},t) = \frac{dz'}{4\pi\varepsilon_\circ}\left\{\cos\theta\left[\frac{2}{r^3}\int_0^t I\left(z',\tau-\frac{r}{c}\right)d\tau + \frac{2}{cr^2}I\left(z',\tau-\frac{r}{c}\right)\right]\boldsymbol{a}_r + \sin\theta\left[\frac{1}{r^3}\int_0^t I\left(z',\tau-\frac{r}{c}\right)d\tau + \frac{1}{cr^2}I\left(z',\tau-\frac{r}{c}\right) + \frac{1}{c^2 r}\frac{\partial I\left(z',\tau-\frac{r}{c}\right)}{\partial t}\right]\boldsymbol{a}_\theta\right\} \quad (13)$$

$$d\boldsymbol{B}(\boldsymbol{r},t) = \frac{\mu_\circ dz'}{4\pi}\sin\theta\left[\frac{1}{r^2}I(z',\tau-r/c) + \frac{1}{cr}\frac{\partial I(z',\tau-r/c)}{\partial t}\right]\boldsymbol{a}_\varphi \quad (14)$$

Where dz' is the 1-D line element of current, **r** and r are the vector and distance from the distance dz' to the observer, t is the time in the observer's frame of reference, θ is the angle measured from dz' to **R**, I is the current magnitude along z' and $\boldsymbol{a_R}$, $\boldsymbol{a_\theta}$ and $\boldsymbol{a_\varphi}$ are the unit vectors in R, θ *and* φ directions in a spherical coordinate frame, respectively.

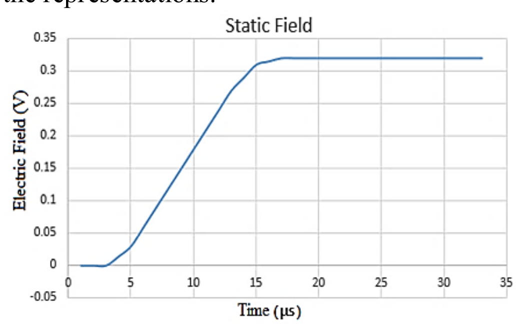
These equations were derived for an infinitely thin, one dimensional source current and not for a general three dimensional current distribution. Hence, for the generalization of these lightning electromagnetic equations (Shao, 2016) introduced a corresponding pair of generalized equations that are determined from the three dimensional time dependent current density distribution based on Jefimenko original electric and magnetic field equations. For this, he made some adjustments so that the Jefimenko electric field equation depends only on the time dependent current density rather than on both the charge and current densities in its original form. According to (Shao, 2016), the generalized three dimensional electric field equation is represented as

$$\boldsymbol{E}(\boldsymbol{x},t) = \frac{1}{4\pi\varepsilon_\circ}\int_0^t d^3x'\left\{\int_0^t \frac{2\left(\boldsymbol{J}\left(\boldsymbol{x}',\tau-\frac{r}{c}\right)\cdot\hat{\boldsymbol{r}}\right)\hat{\boldsymbol{r}} + \left(\left(\boldsymbol{J}\left(\boldsymbol{x}',\tau-\frac{r}{c}\right)\times\hat{\boldsymbol{r}}\right)\times\hat{\boldsymbol{r}}\right)}{r^3}d\tau + \frac{2\left(\boldsymbol{J}\left(\boldsymbol{x}',t-\frac{r}{c}\right)\cdot\hat{\boldsymbol{r}}\right)\hat{\boldsymbol{r}} + \left(\boldsymbol{J}\left(\boldsymbol{x}',t-\frac{r}{c}\right)\times\hat{\boldsymbol{r}}\right)\times\hat{\boldsymbol{r}}}{cr^2} + \frac{\left(\frac{\partial\boldsymbol{J}\left(\boldsymbol{x}',t-\frac{r}{c}\right)}{\partial t}\times\hat{\boldsymbol{r}}\right)\times\hat{\boldsymbol{r}}}{c^2 r}\right\} \quad (15)$$

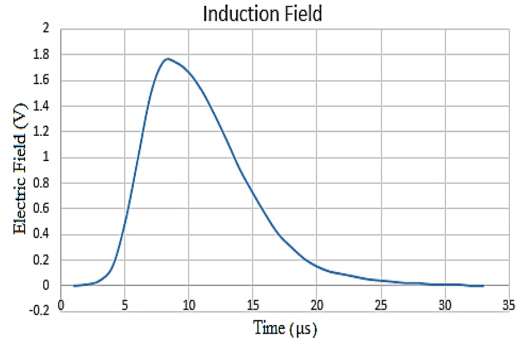
Hence, equation (15) can be considered as the generalized version of equation (13) of Uman et al (1975). Similarly, the derivation of electric field equation of Uman et al., (1975) from the generalized Jefimenko equations by Shao, (2016) further solidifies the flexibility of Jefimenko equations in computations of electric fields of lightning discharges. Hence, from equation (15) the first term containing r^{-3} is electrostatic field, the second term containing r^{-2} is induction field, and the last term containing r^{-1} is radiation field.

5. Discussion and conclusion

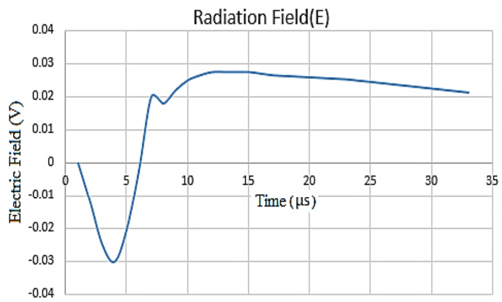
The graphical representation of components of lightning electric field is given in figure 1. The three components of the electric field lightning are described in the figure 1 a, b, c. From figure 1(a), the static component of the electric field increases gradually and remains constant over time. The three dimensional induction field increases to its maximum amplitude and then decreases with increase in time is shown in figure 1(b). The three dimensional radiation field decreases to its minimum value and then gradually increases to its maximum value and then decreases gradually with the increase in time as shown in figure 1(c). The total of these components is given in the figure 1(d). The duration of the static component is the largest in the representations.



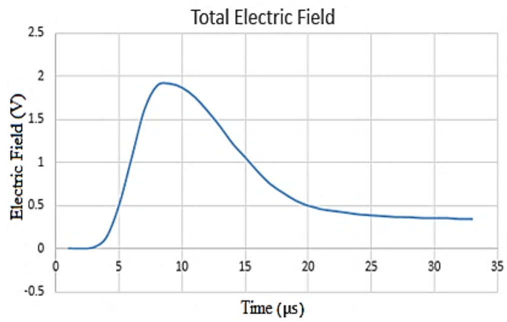
(a)



(b)



(c)



(d)

Figure 1: The three components of the electric field due to lightning are given in the figure 1 (a), (b) and (c); and the total of these components in the figure 1(d).

The individual effect of these components on the total electric field is evaluated using the correlation coefficient between them. The hypothetical data was extracted from the graphical representation of electric field from paper of Thottappillil et al., (2001). The correlation coefficient between the electrostatic component and the total electric field was approximately found to be 0.96, which means that increment in the electrostatic component increases the electric field proportionately as seen in the graph. Similarly, the correlation coefficient between the induction component and the total electric field was found to be 0.76. Hence, the effect of induction component is less when compared to that of the electrostatic field. In contrast, there is a negative correlation between the radiation component and the total electric field (i.e. the correlation coefficient was found to be -0.977). It suggests that decrease in the radiation component increases the total electric field. Hence, from these representations we can conclude that the electrostatic field component dominating for the longer duration and the sum of the electrostatic component and the induction component of the electric field has relatively greater effect to the total electric field than the radiation components in the lightning discharge.

References:

Bazelyan, E. M., & Raizer, Y. P. (2000). *Lightning Physics and Lightning Protection.* london: Institute of Physics Publishing.

Chalmers, J. A. (1965), the so-called "induction" component in the electric field of a lightning discharge. *Journal of Atmospheric and Terrestial Physics*, **27** (9), 1027-1028.

Griffiths, D. J. (1999). *Introduction To Electrodynamics*, New Delhi: Prentice Hall of India 3rd edition.

Heald, M. A., & Griffiths, D. J. (1991), *Time dependent genralization of the Biot-Savart and Coulomb laws, American Journal of Physics,* ***59****, 111-117.*

Lorentz, L. (1867). *On the identity of the vibrations of light with electrical currents,* **34** (230), 287-301.

McDonald, K. T. (1997). *The relation bteween expressions for time-dependent electromagnetic fields given by Jefimenko and by Panofsky and Philips, Princeton University Press,* **60** (11), 1074-1076.

Nickolaenko, A. P., & Hayakawa, M. (1998). Electric fields produced by lightning discharge, *Journal of Geophysical research: atmospheres,* ***103*** (D14), 17175-17189.

Shao, X. M. (2016). *Generalization of the lightning electromagentic equations of Uman, Mclain, and Krieder based on Jefimenko equation,* 121, 3363-3371.

Taylor, W. L. (1963). Rdiation Field Characteristics of Lightning Dischrges in the Band 1kc/s to 100 kc/s, *Journal of Research,* **67D** (No. 5), 539-550.

Thottappillil, R. (2003), *The Lightning Flash* **34**, (V. Cooray, Ed.) London: IEE Press.

Thottappillil, R., & Rakov, V. A. (2001). On different approaches to calculating lightning electric fields. *Journal of GeoPhysical research,* **106** (NO. D13), 14191-14205.

Uman, M. A., Mclain, D. K., & Krider, E. P. (1975), *The electromagnetic radiation from a finite antenna,* **43**, 33-38.